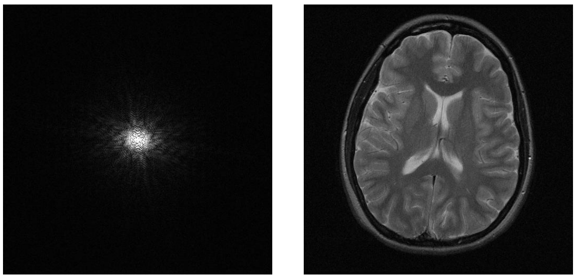
MRI IMAGE RECONSTRUCTION FROM UNDERSAMPLED K-SPACE DATA

Prakhar K.(13485) Satyam Dwivedi(13629)

IIT Kanpur



K-Space Data IFFT Image

Fig. 1. The MR-Image is collected as K-Space data (a). original image can be recovered by IFFT of (a)

ABSTRACT

In this project we explore the paradigm of MRI Reconstruction. MRI scans are collected using Magnetic-Gradient coils, which collect the image data in K-Space domain, which is basically just the Fourier Transform of the original image. Sampling is very time consuming so MR image is reconstructed from undersampled data via Compressed Sensing. We explore Compressed Sensing (CS) in our project. Then we explore different CS-based reconstruction methods.

Index Terms— MRI, K-Space, Compressed Sensing, POCS, SparseMRI, DictMRI

1. INTRODUCTION

MRI images are scans of cross section of human body. MRI images are collected using Magnetic-Gradient coils, which collect the image data in K-Space domain, which is basically just the Fourier Transform of the original image. Collecting these samples requires the patient to stay still for 15-90 minutes, which is often inconvenient. Hence, a technique called Compressed Sensing has been developed for fair reconstruction of MRI image at sub-nyquist sampling rate. We explore CS-based reconstruction techniques in our project.

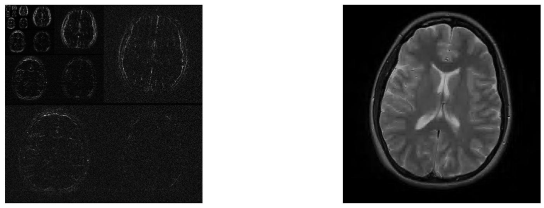
2. COMPRESSED SENSING

2.1. Idea

Compressed Sensing is based on the idea that a randomly undersampled signal is recoverable if it is sparse some other Transform Domain i.e. to be CS-recoverable, a signal has to satisfy two conditions:

- It should be Sparse in some known Transform domain.
- It should be sampled randomly.

Hence instead of sampling the signal at nyquist rate and then compressing it, we directly sense the data in compressed form, hence the name Compressed Sensing [1].



Wavelet Transform Reconstruction using 2% Coeffs

Fig. 2. Wavelet transform of original image and then reconstruction by using just top 2% coefficients

2.2. CS in MRI

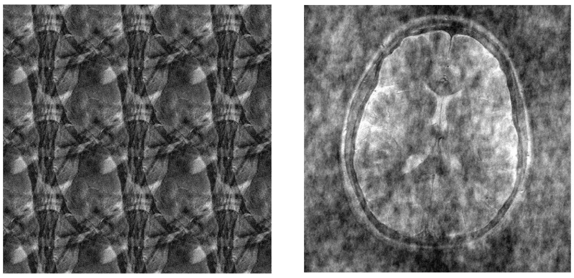
The MRI data is collected in K-Space domain, which is just the FFT of image domain. Now as Fig. 2 suggests, MRI image is sparse in Wavelet Transform Domain.

Also, the IFFT image of Randomly undersampled Fourier Data manifests as random noise in the image domain. However, if the undersampling is not random, then the recovery of signal is not possible (Fig. 3).

The undersampling can be thought of as multiplying the K-Space image with a binary mask of same size. We have used two kinds of mask: uniform random sampling mask, and variable density sampling mask.

2.3. Optimisation Problem

The CS can be mathematically defined as an optimisation problem. Let $\mathbf{m}$ be the image in pixel domain, $\mathbf{y}$ be the collected samples in Fourier domain, $F_u = AF$, where A is the sampling-mask, F is the Fourier-matrix, and let Ψ be the



Uniform undersampling Random Undersampling

Fig. 3. IFFT of Uniform v/s Random undersampling of K-Space data. Signal can be recovered by denoising (b)

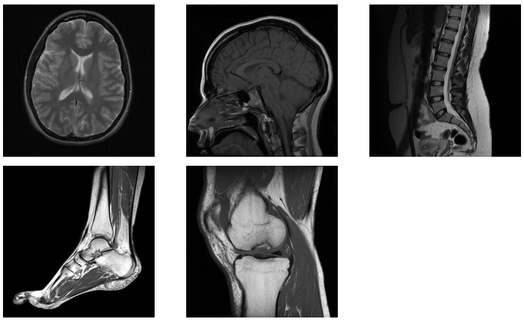


Fig. 4. Test images are (from top left) MRI scans of brain, side-brain, spine, foot and knee

transform domain where m is sparse. Our optimisation problem to get reconstructed signal $\mathbf{m_r}$ is:

$$\mathbf{m_r} = ARGMIN_m \|\Psi \mathbf{m}\|_0 \text{ s.t. } F_u \mathbf{m} = \mathbf{y}$$

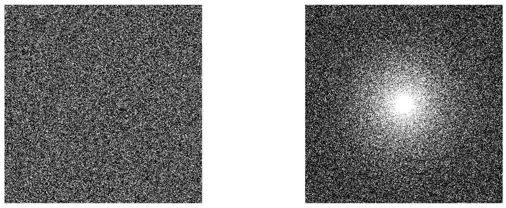
This problem is n.p. hard to solve so we relax the optimisation problem to be:

$$\mathbf{m_r} = ARGMIN_m \{ \|F_u \mathbf{m} - \mathbf{y}\|_2^2 + \lambda \|\Psi \mathbf{m}\|_1 \}$$

3. EXPERIMENTAL SETUP

Michael Lustig has a very nice exercise on CS for MRI on his homepage [2]. We found the exercise very helpful in understanding the basic concepts of CS, and its application in medical imaging. We use 5 images for our methods (Fig. [**?**]). The random subsampling in K-Space domain is simulated by undersampling this K-Space data randomly to retain just one third coefficients. Since most of the energy is concentrated around origin so two undersampling masks (also provided in the exercise) are used: Uniform density mask, and Variable Density (Gaussian) mask (Fig. 5).

The Sparsifying Transform is the Daubechies Wavelet Transform (DWT Toolbox given with the exercise). We have also used K-SVD toolbox ([3]). We have used author's code for SparseMRI [4].



uniform Variable Density

Fig. 5. Random Sampling masks

4. RECONSTRUCTION METHODS

4.1. Projection onto Convex Sets (POCS)

The POCS method is an iterative solution of the CS Optimisation problem. We apply soft threshold in Wavelet Domain and then enforce data-consistency in Fourier Domain. The algorithm can be formulated as (details in [2]):

- Initialise $\mathbf{y_r} = \mathbf{y}$ and $\mathbf{m_r}$ and then repeat until convergence (more details in [2]):
- $\mathbf{m_r} = IFFT(\mathbf{y_r})$
- Take DWT of $\mathbf{m_r}$, soft-threshold all coefficients by λ, take IDWT and store in $\mathbf{m_r}$.
- $\mathbf{y_r} = FFT(\mathbf{m_r})$ and enforce data consistency (non-zero coefficients of $\mathbf{y}$ are forced-set into $\mathbf{y_r}$)

4.2. Non-Linear Conjugate Gradient Descent with Back-Tracking Line Search (SparseMRI)

SparseMRI[4] modifies the original problem statement to include Finite Differences also i.e. enforce sparsity in both DWT domain as well as in Finite Differences domain (FD):

$$\mathbf{m_r} = ARGMIN_m \{ \|F_u \mathbf{m} - \mathbf{y}\|_2^2 + \lambda \|\Psi \mathbf{m}\|_1 + \alpha TV(\mathbf{m}) \}$$

where Total Variation is $TV(\mathbf{m}) = \|FD(\mathbf{m})\|_1$. Since MRI is a continuous and smooth image so it should be sparse in TV domain. They solve this optimisation problem using Non-Linear Conjugate Gradient Descent (NLCGD) with Back-Tracking line search [details in [4]].

4.3. Adaptive Dictionary for MRI (DictMRI)

ADL[5] basically uses an overcomplete dictionary of image-patches as the sparse domain. The dictionary is learnt by extracting patches from the image. The Optimisation Problem can be formulated as

$$\min_{\mathbf{m}, D, \Gamma} \sum_{i,j} \|R_{ij} \mathbf{m} - D\alpha_{ij}\| + \nu \|F_u \mathbf{m} - \mathbf{y}\|_2^2$$

given

$$\|\alpha_{ij}\|_0 \leq T_0 \forall i, j$$

where $R_{ij}x$ is the $(i, j)^{th}$ patch, α_{ij} is its sparse projection via Dictionary D. Image $\mathbf{m}$, Dictionary D and α_{ij}(s) are learnt in this algorithm.

The Initialisation is $\mathbf{y_r} = \mathbf{y}$ and $\mathbf{m_r}$ and then following steps are repeated repeat until convergence:

- Extract patches from image
- Learn Dictionary D over a random subset of these patches using K-SVD.
- Obtain the sparse vectors α_{ij} for each patch using OMP.
- Reconstruct all the patches and combine these patches to create a modified image.
- Obtain the FFT of this modified image and restore the Original K-Space coefficients.
- Take IFFT to obtain the image.

Basically we iterate between learning Dictionary with reconstructed patches via K-SVD, and then combining the patches followed by enforcing data consistency in Fourier Domain (details in [5]). The time complexity of various steps of this algorithm are given in [5]:

- K-SVD: $\mathcal{O}(\delta N K n T_0 J)$, $\delta J \approx 1$
- OMP: $\mathcal{O}(N K n T_0)$
- FFT and IFFT: $\mathcal{O}(P \log P)$

Here, N: No. of patches, δ: Fraction, K: No. of Dict. atoms, n: Patch size, T_0: Sparsity, J: iterations in K-SVD, P: Image size. It can be easily seen that K-SVD and OMP are the bottleneck. We try to improve this method.

5. RESULTS AND INFERENCE

We'll evaluate our reconstructed images by obtain the RMSE and Structural Similarity Index (SSIM) w.r.t. the Original image. A good reconstruction will have higher SSIM, and lower RMSE. First we'll observe reconstruction error for variable density mask and uniform random mask. Results will show that for same undersampling factor, variable density mask is preferable. Then we'll show experiments using different methods for variable density masks.

Image	Unif	VarDens
Brain	0.0232	0.0018
Brain(s)	0.0245	0.0004
Spine	0.0441	0.0006
Foot	0.0646	0.0031
Knee	0.0498	0.0004

Table 1. Reconstruction evaluation for Unif. and VarDens Sampling

5.1. IFFT Reconstruction

The simplest way to reconstruct the images is simply taking the IFFT of the undersampled K-Space data. The undersampled coefficients are upscaled by inverse of their probability, to compensate for the energy of unsampled coefficients. For uniform sampling mask, the probability of getting sampled for each coefficient is equal. For variable density mask, prob. is higher for coefficients which are closer to the Origin, which makes sense since most of the coefficients are concentrated near origin so more samples should be take near it. The results are given in table 1. Note that for each mask finally samples only 33% of coefficients. Yet results for variable density mask are much better. Hence, we'll use this mask only for further experiments (in real life setting, one can generate a VarDens mask and then take samples at only those locations where VarDens(i,j) = 1).

5.2. Results for CS-based algorithms

As we said we have tried three algorithms: POCS, SparseMRI, and DictMRI. For SparseMRI, we used author's code and their own parameter settings. POCS and DictMRI were implemented by ourselves. We tried these algorithms for 2 different undersampling factors of 3.0 and 8.5. The results for undersampling factor of 3.0 are given in Table 2 and for factor of 8.5 in Table 3. The results for POCS here are far better than those given in presentation, because we changed a step in the algorithm. Earlier we were enforcing data consistency using scaled coefficients i.e. Force (i, j)th coefficient to always be $Y(i, j)/p(i, j)$, if it was sampled. However (i, j)th coefficient should just be $Y(i, j)$. Correcting this step led to increase in performance. We are using 50 iterations, and $\lambda = 0.01$.

For the DictMRI algorithm, we took the patch-size to be 6x6, stride 3, and the dictionary size as 36x36. Random patches to learn K-SVD were set to 10k. Increasing the dictionary size doesn't seem to have much effect. Also, we tried three different methods of initialising the dictionary:

- **Learnt Dictionary**: We learnt the dictionary on 3 images and used it as initialising dictionary for other 2 images.
- **K-Means Initialisation**: We perform K-Means with

Image	IFFT	POCS	SparseMRI	DictMRI
Brain	0.0018	0.0007	0.0006	0.0065
Brain(s)	0.0004	8.1e-05	0.0001	0.0001
Spine	0.0006	0.0001	0.0001	0.0002
Foot	0.0031	0.0009	0.0001	0.0009
Knee	0.0004	0.0001	0.0002	0.0002

Image	IFFT	POCS	SparseMRI	DictMRI
Brain	0.5777	0.7650	0.7437	0.6647
Brain(s)	0.7999	0.9618	0.9716	0.9536
Spine	0.6888	0.9404	0.9593	0.9174
Foot	0.3633	0.8280	0.9833	0.6388
Knee	0.8105	0.9618	0.9620	0.9490

Table 2. Evaluation of RMSE (upper table) and SSIM (lower table) for CS-based algorithms for undersampling factor of 3

K=36 on the random patches, and use centers of all clusters as initialising columns of dictionary

- **Correlation Based Initialisation**: We compute pairwise distances between all random 10k patches and choose 36 most distant patches as columns.

Out of the three methods K-Means was performing best so we took K-Means method for all subsequent experiments. The RMSE, SNR and SSIM variation for the 1st MRI image for DictMRI is given in Fig 6. The SNR decays and then remains constant. So the performance seems bad. However, for the undersampling by 8.5 case, SNR keeps increasing, RMSE keeps decreasing and SSIM also keeps increasing (Fig 7). But still the overall performance of DictMRI is coming out to be inferior to other two methods, despite taking more time and memory to execute.

The average time taken by all the algorithms is:

- POCS: 32.1s for 50 iterations
- SparseMRI: 118.9s for 15 iterations
- DictMRI: 789.5s for 50 iterations

6. CONCLUSION

We explored the paradigm of Compressed Sensing in the image-modality of MRI. CS-based algorithms are very effective in recovering MRI images. Out of the three methods we saw, according to our experiments, the winner in terms of performance is clearly SparseMRI, but POCS takes around one fourth time and still gives comparable results. Results of DictMRI are also acceptable, they are far better than simple inverse FFT, but we had hoped for it to easily beat other algorithms by a good margin. We tried hard and improved it a little bit but still it didn't perform as per our expectations. We conclude with Fig. 8 comparing the reconstructions of these methods.

Image	IFFT	POCS	SparseMRI	DictMRI
Brain	0.0150	0.0014	0.0020	0.0015
Brain(s)	0.0324	0.0004	0.0002	0.0003
Spine	0.0027	0.0004	0.0003	0.0004
Foot	0.0080	0.0043	0.0018	0.0017
Knee	0.0010	0.0009	0.0004	0.0004

Image	IFFT	POCS	SparseMRI	DictMRI
Brain	0.1583	0.6871	0.6742	0.6177
Brain(s)	0.3207	0.9329	0.9376	0.9185
Spine	0.4542	0.8933	0.9119	0.8940
Foot	0.2935	0.7332	0.8826	0.7048
Knee	0.6693	0.9153	0.9214	0.8974

Table 3. Evaluation of RMSE (upper table) and SSIM (lower table) for CS-based algorithms for undersampling factor of 8.5

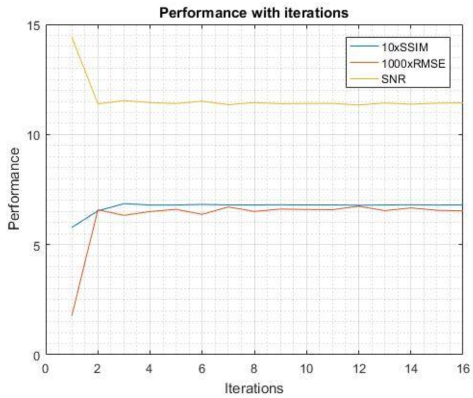


Fig. 6. SNR, RMSE, SSIM variation for DictMRI for undersampling factor of 3

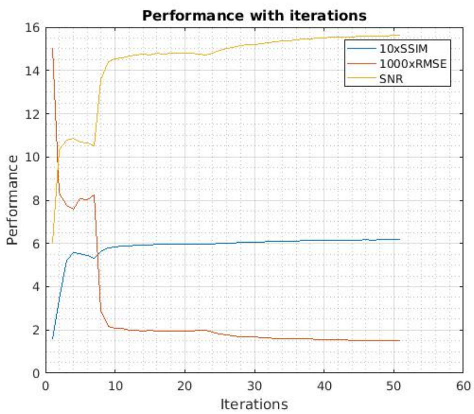


Fig. 7. SNR, RMSE, SSIM variation for DictMRI for undersampling factor of 8.5

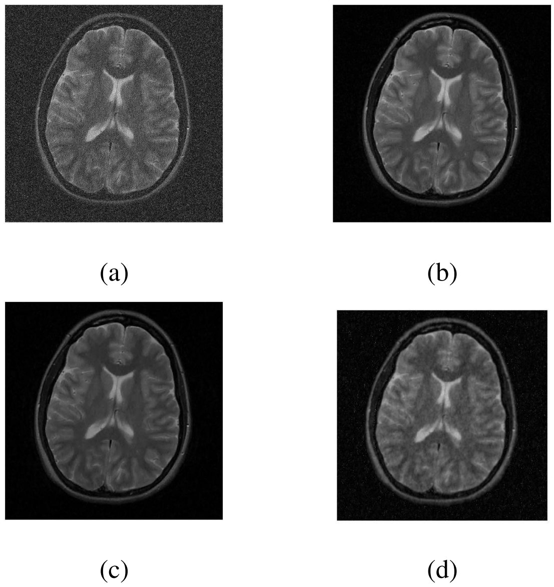


Fig. 8. IFFT of Brain MRI K-Space undersampled by 8.5 (a) and its Reconstruction by POCS (b), SparseMRI (c), DictMRI (d)

7. REFERENCES

[1] Duarte M. F. Eldar Y. C. Kutyniok G. Davenport, M. A., *Introduction to compressed sensing.*

[2] Michael Lustig., "Cs exercise on mri, ee369c medical image reconstruction, autumn 2007, university of california, berkeley.," *link: http://people.eecs.berkeley.edu/ mlustig/CS.html*.

[3] Ron Rubinstein., "Matlab software packages," *link: http://www.cs.technion.ac.il/ ronrubin/software.html*.

[4] David Donoho Lustig, Michael and John M. Pauly., "Sparse mri: The application of compressed sensing for rapid mr imaging.," *Magnetic resonance in medicine 58.6 (2007): 1182-1195.*

[5] Saiprasad Ravishankar and Yoram Bresler., "Mr image reconstruction from highly undersampled k-space data by dictionary learning.," *IEEE transactions on medical imaging 30.5 (2011): 1028-1041.*