MRI Image Reconstruction from Undersampled K-Space data

EE698K Course Project

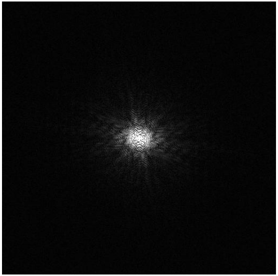
Prakhar K. (13485)[1], Satyam Dwivedi (13629)[1]

[1]Dept. of EE, IIT Kanpur

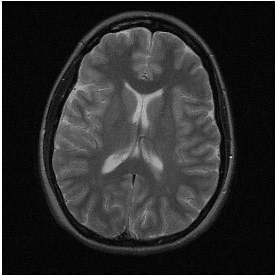
Instructor: Prof. Tanaya Guha

Outline

- MRI scans are collected using Magnetic-Gradient coils, which collect the image data in K-Space domain, which is basically just the Fourier Transform of the original image.



K-Space Data



IFFT Image

Introduction

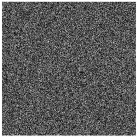
- Collecting these samples requires the patient to stay still for 15-90 minutes, which is often inconvenient.
- Collection time can be reduced by reducing the no. of samples collected.
- Techniques have been developed for fair reconstruction of MRI image at sub-nyquist sampling rate.

Idea

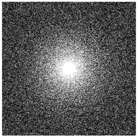
Compressed Sensing

- It is possible to reconstruct an undersampled signal, if the sampling was random.
- Random undersampling in K-Space creates noise like aliasing in the image domain i.e. removing aliasing is similar to denoising.
- The requirement is that the signal must be Sparse in some Transform domain.
- Enforcing sparsity in that domain should result in recovery of unsampled coefficients.

- In case of MRI, sampling is done in K-Space (Fourier Domain). Sampling can be uniform or variable density:

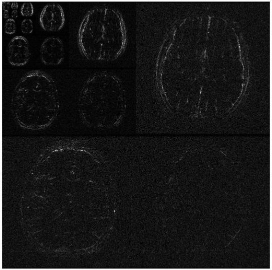


Uniform Sampling

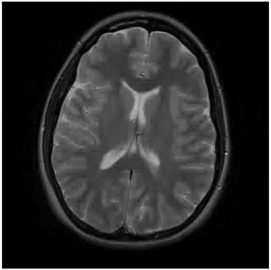


Variable Density Sampling

- MRI image is sparse in eg: Wavelet-Domain.



Wavelet Transform



Reconstruction using 2% Coeffs

Optimisation Problem

Compressed Sensing

Let **m** be the image in pixel domain, **y** be the collected samples in Fourier domain, $F_u = AF$, where A is the sampling-mask, F is the Fourier-matrix, and let Ψ be the transform domain where m is sparse. Our optimisation problem to get reconstructed signal $\mathbf{m_r}$ is:

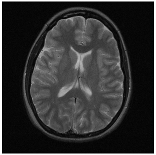
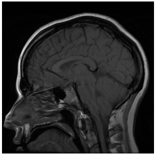
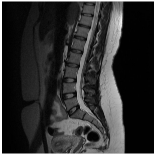
$$\mathbf{m_r} = ARGMIN_m \|\Psi \mathbf{m}\|_0 \text{ s.t. } F_u \mathbf{m} = \mathbf{y}$$

This problem is n.p. hard to solve so we relax the optimisation problem to be:

$$\mathbf{m_r} = ARGMIN_m \{\|F_u \mathbf{m} - \mathbf{y}\|_2^2 + \lambda \|\Psi \mathbf{m}\|_1\}$$

Experimental Setup

- We use 5 512x512 MRI images, and take their FFT.

- Then we apply sampling masks to simulated undersampled K-Space data.
- Then we try our methods on this data.

Outline

- Initialise $\mathbf{y_r} = \mathbf{y}$ and $\mathbf{m_r}$ and then repeat until convergence (more details in [2]):
  - $\mathbf{m_r} = IFFT(\mathbf{y_r})$
  - Take DWT of $\mathbf{m_r}$, soft-threshold all coefficients by λ, take IDWT and store in $\mathbf{m_r}$.
  - $\mathbf{y_r} = FFT(\mathbf{m_r})$ and enforce data consistency (non-zero coefficients of $\mathbf{y}$ are forced-set into $\mathbf{y_r}$)

Outline

- SparseMRI[1] modifies the original problem statement to include Finite Differences also i.e. enforce sparsity in both DWT domain as well as in Finite Differences domain (FD):

$$\mathbf{m_r} = ARGMIN_m\{\|F_u\mathbf{m} - \mathbf{y}\|_2^2 + \lambda\|\Psi\mathbf{m}\|_1 + \alpha TV(\mathbf{m})\}$$

  where Total Variation is $TV(\mathbf{m}) = \|FD(\mathbf{m})\|_1$. They solve this optimisation problem using Non-Linear Conjugate Gradient Descent (NLCGD) with Back-Tracking line search [details in [1]].
- We have used the author's implementation on our images and masks, for this method.

Outline

- ADL[3] basically uses an overcomplete dictionary of image-patches as the sparse domain.
- The dictionary is learnt by extracting patches from the image.
- Optimisation Problem:

$$\min_{\mathbf{m},D,\Gamma} \sum_{i,j} \|R_{ij}\mathbf{m} - D\alpha_{ij}\| + \nu\|F_u\mathbf{m} - \mathbf{y}\|_2^2$$

given

$$\|\alpha_{ij}\|_0 \leq T_0 \forall i,j$$

where $R_{ij}x$ is the $(i,j)^{th}$ patch, α_{ij} is its sparse projection via Dictionary D.

- Image **m**, Dictionary D and α_{ij}(s) are learnt.

- Initialise $\mathbf{y_r} = \mathbf{y}$ and $\mathbf{m_r}$ and then repeat until convergence:
- Extract patches from image
- Learn Dictionary D over a random subset of these patches using K-SVD.
- Obtain the sparse vectors α_{ij} for each patch using OMP.
- Reconstruct all the patches and combine these patches to create a modified image.
- Obtain the FFT of this modified image and restore the Original K-Space coefficients.
- Take IFFT to obtain the image.

- The time-complexities for steps are:
  - K-SVD: $\mathcal{O}(\delta N K n T_0 J)$, $\delta J \approx 1$
  - OMP: $\mathcal{O}(N K n T_0)$
  - FFT and IFFT: $\mathcal{O}(P \log P)$
- N: No. of patches, δ: Fraction, K: No. of Dict. atoms, n: Patch size, T_0: Sparsity, J: iterations in K-SVD, P: Image size.
- K-SVD and OMP are the bottleneck.
- We try to improve the ADA method.

Adaptive Dictionary for MRI

Reconstruction Methods

- We are trying some modifications to improve time/performance.
- The patches with extremely low average intensities are directly assigned alpha vector $\mathbf{0}$.
- Dictionary initialisation is a very important step in K-SVD. paper uses K random patches to initialise K atoms.
- Along with this method we also initialise dictionary by centroids of K-Means over patches, and by a dictionary learnt on patches of other MR images.

- We observe RMSE for direct FFT of undersampled data for 2 masks:

Image	Unif	Vardens
Brain	0.0232	0.0018
Brain(s)	0.0245	0.0004
Spine	0.0441	0.0006
Foot	0.0646	0.0031
Knee	0.0498	0.0004

- It is clear that Variable Density mask is better.

- We observe RMSE for the algorithms we have used:

Image	IFFT	POCS	SparseMRI	DictMRI
Brain	0.0018	0.0007	0.0006	0.0065
Brain(s)	0.0004	8.1e-05	0.0001	0.0001
Spine	0.0006	0.0001	0.0001	0.0002
Foot	0.0031	0.0009	0.0001	0.0009
Knee	0.0004	0.0001	0.0002	0.0002

- POCS is performing worse than simple IFFT, while SparseMRI is giving good results.

Structural Similarity Index (SSIM)

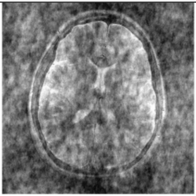
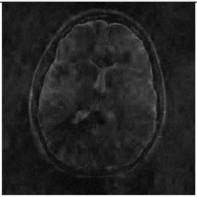
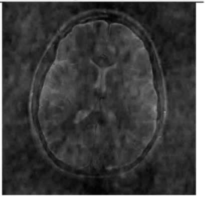
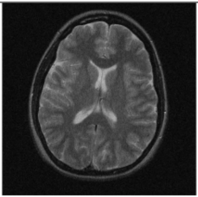
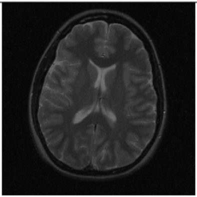
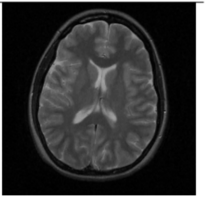
Results

- We observe SSIM for the algorithms we have used:

Image	IFFT	POCS	SparseMRI	DictMRI
Brain	0.5777	0.7650	0.7437	0.6647
Brain(s)	0.7999	0.9618	0.9716	0.9536
Spine	0.6888	0.9404	0.9593	0.9174
Foot	0.3633	0.8280	0.9833	0.6388
Knee	0.8105	0.9618	0.9620	0.9490

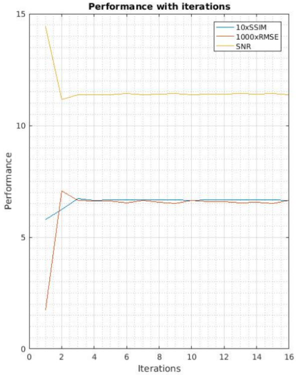
- Here POCS is performing better than simple IFFT, while SparseMRI is performing the best.

Results on some methods
Results

	Inverse FFT	POCS	SparseMRI
Unif Sampled			
RMSE	0.0232	0.0187	0.0168
SSIM	0.2571	0.3498	0.4466
VarDens Sampled			
RMSE	0.0018	0.0062	0.0006
SSIM	0.5777	0.6785	0.7441

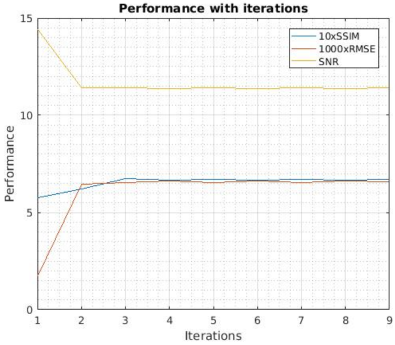
Results on Adaptive Dictionary
Results

- We decided to write our own code here because the author's code wasn't working on our PCs.
- We first learnt a dictionary of size 36x72 from 6k 6x6 random patches from 3 different MR Images, and fixed it in the DL step.

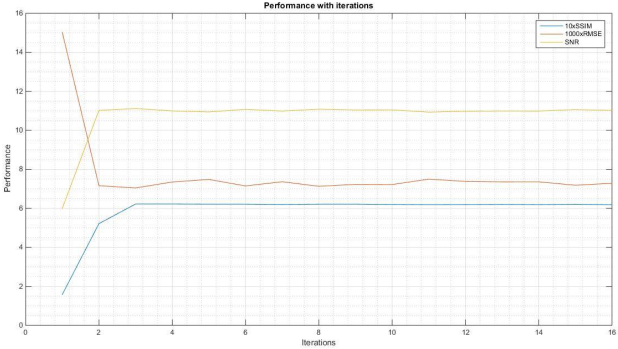


- Results were bad.

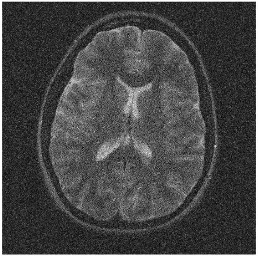
- We also tried learning the dictionary (initialised with the learnt dictionary) from random patches via K-SVD .

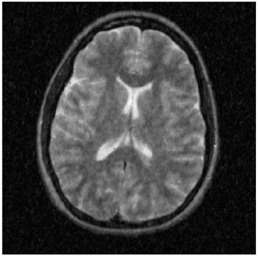


- Paper stated that the main strength of ADMRI is its ability to work for highly undersampled data. Following is the result for undersamping factor of 8.5.



- Reconstructed image from undersampling factor of 8.5 is as follows:





- Then we initialised the Dictionary with K-Means centroid over the data. After that it is sent into K-SVD.
- We also replaced K-SVD with Method Of Optimal Dictionary (MOD).
- Initialization using a Dictionary learnt on training images was also tried.
- In all the cases, our results are very inferior to the SparseMRI, but the results of paper are better than SparseMRI.

Conclusion

- We have seen three different methods of Image-Reconstruction.
- The Adaptive Dictionary based method has to be improved. Problem in most probably in the Dictionary Learning step.
- We tried various modifications to the original Adaptive Dictionary method, to improve our results.
- In all the cases, our results are very inferior to the SparseMRI.

References I

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